The Parma Polyhedra Library OCaml Language Interface User's Manual* (version 0.10)

Roberto Bagnara†
Patricia M. Hill‡
Enea Zaffanella§

November 4, 2008

*This work has been partly supported by: University of Parma's FIL scientific research project (ex 60%) "Pure and Applied Mathematics"; MURST project "Automatic Program Certification by Abstract Interpretation"; MURST project "Abstract Interpretation, Type Systems and Control-Flow Analysis"; MURST project "Automatic Aggregate- and Number-Reasoning for Computing: from Decision Algorithms to Constraint Programming with Multisets, Sets, and Maps"; MURST project "Constraint Based Verification of Reactive Systems"; MURST project "Abstract Interpretation: Design and Applications"; EPSRC project "Numerical Domains for Software Analysis"; EPSRC project "Geometric Abstractions for Scalable Program Analyzers".

†bagnara@cs.unipr.it, Department of Mathematics, University of Parma, Italy.
‡hill@comp.leeds.ac.uk, School of Computing, University of Leeds, U.K.
§zaffanella@cs.unipr.it, Department of Mathematics, University of Parma, Italy.

For the most up-to-date information see the Parma Polyhedra Library site:

`http://www.cs.unipr.it/ppl/`

Contents

1 OCaml Language Interface

The Parma Polyhedra Library comes equipped with an interface for the OCaml language.

The main features of the library are described in Section OCaml Interface Features. Section OCamldoc Documentation lists all the functions available to the default generated domains in the OCaml interface. Section Compilation and Installation explains how the OCaml interface is compiled and installed.

In the sequel, `prefix` is the prefix under which you have installed the library (typically `/usr` or `/usr/local`).

OCaml Interface Features

The OCaml interface provides access to the numerical abstractions (convex polyhedra, BD shapes, octagonal shapes, etc.) implemented by the PPL library. A general introduction to the numerical abstractions, their representation in the PPL and the operations provided by the PPL is given in the main *PPL user manual*. Here we just describe those aspects that are specific to the OCaml interface.

Overview

First, here is a list of notes with general information and advice on the use of the OCaml interface.

- The numerical abstract domains available to the OCaml user consist of the *simple* domains, *powersets* of a simple domain and *products* of simple domains.
  - The simple domains are:
    - convex polyhedra, which consist of C_Polyhedron and NNC_Polyhedron;
    - weakly relational, which consist of BD_Shape_N and Octagonal_Shape_N where N is one of the numeric types short, signed_char, int, long, long_long, mpz_class, mpq_class;
    - boxes which consist of Int8_Box, Int16_Box, Int32_Box, Int64_Box, Uint8_Box, Uint16_Box, Uint32_Box, Uint64_Box, Double_Box, Long_Double_Box, Z_Box, Rational_Box, Float_Box; and
    - the Grid domain.
  - The powerset domains are Pointset_Powerset_S where S is a simple domain.
  - The product domains consist of Direct_Product_S_T, Smash_Product_S_T and Constraints_-Product_S_T where S and T are simple domains.
- In the following, any of the above numerical abstract domains is called a PPL *domain* and any element of a PPL domain is called a *PPL object*.
- The OCaml interface files are all installed in the directory `prefix/lib/ppl`. Since this includes shared and dynamically loaded libraries, you must make your dynamic linker/loader aware of this fact. If you use a GNU/Linux system, try the commands `man ld.so` and `man ldconfig` for more information.
- A PPL object such as a polyhedron can only be accessed by means of a OCaml term called a *handle*. Note, however, that the data structure of a handle, is implementation-dependent, system-dependent and version-dependent, and, for this reason, deliberately left unspecified. What we do guarantee is that the handle requires very little memory.
- An OCaml program can obtain a valid handle for a PPL object by using functions such as

  ```
  ppl_new_C_Polyhedron_from_space_dimension,
  ppl_new_C_Polyhedron_from_C_Polyhedron,
  ppl_new_C_Polyhedron_from_constraints,
  ppl_new_C_Polyhedron_from_generators,
  ```

  These functions will return a new handle for referencing a PPL polyhedron.
- For a PPL object with space dimension `k`, the identifiers used for the PPL variables must lie between 0 and $k-1$ and correspond to the indices of the associated Cartesian axes. For example, when using the functions that combine PPL polyhedra or add constraints or generators to a representation of a PPL polyhedron, the polyhedra referenced and any constraints or generators in the call should follow all the (space) dimension-compatibility rules stated in Section *Representations of Convex Polyhedra* of the main PPL user manual.

- As explained above, a polyhedron has a fixed topology C or NNC, that is determined at the time of its initialization. All subsequent operations on the polyhedron must respect all the topological compatibility rules stated in Section *Representations of Convex Polyhedra* of the main PPL user manual.
- Any application using the PPL should make sure that only the intended version(s) of the library are ever used. Functions

```
ppl_version_major/1,
ppl_version_minor/1,
ppl_version_revision/1,
ppl_version_beta/1,
ppl_version/1,
ppl_banner.
```

  allow run-time checking of information about the version being used.

Function Descriptions

Below is a short description of many of the interface functions. For full definitions of terminology used here, see the main PPL user manual.

1.0.0.1 * Domain Independent Functions

First we describe some domain independent functions included with all instantiations of the OCaml interfaces.

`ppl_version_major`

Returns the major number of the PPL version.

`ppl_version_minor`

Returns the minor number of the PPL version.

`ppl_version_revision`

Returns the revision number of the PPL version.

`ppl_version_beta`

Returns the beta number of the PPL version.

`ppl_version`

Returns the PPL version.

`ppl_banner`

Returns information about the PPL version, the licensing, the lack of any warranty whatsoever, the C++ compiler used to build the library, where to report bugs and where to look for further information.

`ppl_Coefficient_is_bounded`

Returns true if and only if the Coefficients in the C++ interface are bounded.

`ppl_Coefficient_max`

Returns the maximum coefficient the C++ interface can handle.

`ppl_Coefficient_min`

Returns the minimum coefficient the C++ interface can handle.

`ppl_max_space_dimension`

Returns the maximum space dimension this library can handle.

`ppl_set_timeout_exception_atom name`

Sets the term to be thrown by timeout exceptions to `name`. The default value is `time_out`.

`ppl_timeout_exception_atom`

Returns the name to be thrown by timeout exceptions

`ppl_set_timeout c_unsigned`

Computations taking exponential time will be interrupted some time after `c_unsigned` ms after that call. If the computation is interrupted that way, the current timeout exception atom will be thrown. `c_unsigned` must be strictly greater than zero.

`ppl_reset_timeout`

Resets the timeout time so that the computation is not interrupted.

1.0.0.2 * MIP Functions

Here we describe some functions available for PPL objects defining mixed integer (linear) programming problems.

`ppl_new_MIP_Problem_from_space_dimension dimension`

Return a handle to an MIP Problem MIP with the feasible region the vector space of dimension `dimension`, objective function 0 and optimization mode `max`.

`ppl_new_MIP_Problem constraint_system lin_expr optimization_mode`

Return a handle to an MIP Problem MIP with the feasible region represented by `constraint_system`, objective function `lin_expr` and optimization mode `optimization_mode`.

`ppl_MIP_Problem_swap handle_1 handle_2`

Swaps the MIP Problem referenced by `handle_1` with the one referenced by `handle_2`.

`ppl_MIP_Problem_space_dimension handle`

Returns the dimension of the vector space in which the MIP Problem referenced by `handle` is embedded.

`ppl_MIP_Problem_integer_space_dimensions handle`

Returns a list of variables representing representing the integer space dimensions of the MIP Problem referenced by `handle`.

`ppl_MIP_Problem_constraints handle`

Returns a list of the constraints in the constraints system representing the feasible region for the MIP Problem referenced by `handle`.

`ppl_MIP_Problem_objective_function handle`

Returns the objective function for the MIP Problem referenced by `handle`.

`ppl_MIP_Problem_optimization_mode handle`

Returns the optimization mode for the MIP Problem referenced by `handle`.

`ppl_MIP_Problem_clear handle`

Resets the MIP problem referenced by `handle` to be the trivial problem with the feasible region the 0-dimensional universe, objective function 0 and optimization mode `Maximization`.

`ppl_MIP_Problem_add_space_dimensions_and_embed handle dimension`

Embeds the MIP problem referenced by `handle` in a space that is enlarged by `dimension` dimensions,

`ppl_MIP_Problem_add_to_integer_space_dimensions handle vars_list`

Updates the MIP Problem referenced by `handle` so that the variables in `vars_list` are added to the set of integer space dimensions.

`ppl_MIP_Problem_add_constraint handle constraint`

Updates the MIP Problem referenced by `handle` so that the feasible region is represented by the original constraint system together with the constraint `constraint`.

`ppl_MIP_Problem_add_constraints handle constraint_system`

Updates the MIP Problem referenced by `handle` so that the feasible region is represented by the original constraint system together with all the constraints in `constraint_system`.

`ppl_MIP_Problem_set_objective_function handle lin_expr`

Updates the MIP Problem referenced by `handle` so that the objective function is changed to `lin_expr`.

`ppl_MIP_Problem_set_optimization_mode handle optimization_mode`

Updates the MIP Problem referenced by `handle` so that the optimization mode is changed to `optimization_mode`.

`ppl_MIP_Problem_is_satisfiable handle`

Returns true if the MIP Problem referenced by `handle` is satisfiable and false otherwise.

`ppl_MIP_Problem_solve handle`

Solves the MIP problem referenced by `handle` and returns 0, if the MIP problem is not satisfiable; 1, if the MIP problem is satisfiable but there is no finite bound to the value of the objective function; 2, if the MIP problem admits an optimal solution.

`ppl_MIP_Problem_feasible_point handle`

Returns a feasible point for the MIP problem referenced by `handle`.

`ppl_MIP_Problem_optimizing_point handle`

Returns an optimizing point for the MIP problem referenced by `handle`.

`ppl_MIP_Problem_optimal_value handle`

Returns a pair of numbers, the first being the numerator and the second the denominator, for the optimal value for the MIP problem referenced by `handle`.

`ppl_MIP_Problem_evaluate_objective_function handle generator`

Evaluates the objective function of the MIP problem referenced by `handle` at point `generator`. Returns a pair of numbers, the first being the numerator and the second the denominator, for the objective function value for the MIP problem referenced by `handle`.

`ppl_MIP_Problem_OK handle`

Returns true if the MIP Problem referenced by `handle` is well formed, i.e., if it satisfies all its implementation invariants and false, otherwise. Useful for debugging purposes.

1.0.0.3 * C_Polyhedron Functions

Here we describe the main functions available for PPL objects defining convex and closed polyhedra. A list of the available PPL functions for convex polyhedra for the OCaml interface is given in Section OCamldoc Documentation.

`ppl_new_C_Polyhedron_from_space_dimension space_dimension universe_-or_empty`

Returns a handle to a C polyhedron $\mathcal{P}$ with `space_dimension` dimensions; it is empty or the universe polyhedron depending on whether `universe_or_empty` is `empty` or `universe`, respectively.

`ppl_new_C_Polyhedron_from_C_Polyhedron handle`

If `handle` refers to a C polyhedron $\mathcal{P}_1$, then the expression will returns a handle to a copy $\mathcal{P}_2$ of $\mathcal{P}_1$.

`ppl_new_C_Polyhedron_from_NNC_Polyhedron handle`

If `handle` refers to an NNC polyhedron $\mathcal{P}_1$, then the expression returns a handle to a copy $\mathcal{P}_2$ of $\mathcal{P}_1$.

When using `ppl_new_C_Polyhedron_from_NNC_Polyhedron/2`, care must be taken that the source polyhedron referenced by `handle` is topologically closed.

`ppl_new_C_Polyhedron_from_constraints constraint_system`

Returns a handle to a C polyhedron $\mathcal{P}$ represented by `constraint_system`.

`ppl_new_C_Polyhedron_from_generators generator_system`

Returns a handle to a C polyhedron $\mathcal{P}$ represented by `generator_system`.

`ppl_Polyhedron_swap handle_1 handle_2`

Swaps the polyhedron $\mathcal{P}$ referenced by `handle_1` with the polyhedron $\mathcal{Q}$ referenced by `handle_2`. The polyhedra $\mathcal{P}$ and $\mathcal{Q}$ must have the same topology.

`ppl_Polyhedron_space_dimension handle`

Returns the dimension of the vector space in which the polyhedron referenced by `handle` is embedded.

`ppl_Polyhedron_affine_dimension handle`

Returns the actual dimension of the polyhedron referenced by `handle`.

`ppl_Polyhedron_get_constraints handle`

Return a list of the constraints in the constraints system representing the polyhedron referenced by `handle`.

`ppl_Polyhedron_get_minimized_constraints handle`

Returns a minimized list of the constraints in the constraints system representing the polyhedron referenced by `handle`.

`ppl_Polyhedron_get_generators handle`

Returns a list of the generators in the generators system representing the polyhedron referenced by `handle`.

`ppl_Polyhedron_get_minimized_generators handle`

Returns a minimized list of the generators in the generators system representing the polyhedron referenced by `handle`.

`ppl_Polyhedron_relation_with_constraint handle constraint`

Returns the list of relations the polyhedron referenced by `handle` has with `constraint`. The possible relations and their meaning is given in Section *Relation-With Operators* of the main PPL user manual.

`ppl_Polyhedron_relation_with_generator handle generator`

Returns the list of relations the polyhedron referenced by `handle` has with `generator`. The possible relations and their meaning is given in Section *Relation-With Operators* of the main PPL user manual.

`ppl_Polyhedron_is_empty handle`

Returns true if the polyhedron referenced by `handle` is empty and false, otherwise.

`ppl_Polyhedron_is_universe handle`

Returns true if the polyhedron referenced by `handle` is the universe and false, otherwise.

`ppl_Polyhedron_is_bounded handle`

Returns true if the polyhedron referenced by `handle` is bounded and false, otherwise.

`ppl_Polyhedron_contains_integer_point handle`

Returns true if the polyhedron referenced by `handle` contains at least one integer point and false, otherwise.

`ppl_Polyhedron_bounds_from_above handle lin_expr`

Returns true if the polyhedron referenced by `handle` is bounded from above by `lin_expr` and false, otherwise.

`ppl_Polyhedron_bounds_from_below handle lin_expr`

Returns true if the polyhedron referenced by `handle` is bounded from below by `lin_expr` and false, otherwise.

`ppl_Polyhedron_maximize handle lin_expr`

Returns a record `bool_1 * coefficient_1 * coefficient_2 * bool_2` where: `bool_1` is true if the polyhedron P referenced by `handle` is not empty and `lin_expr` is bounded from above in P and false, otherwise. `coefficient_1` is the numerator of the supremum value and `coefficient_2` the denominator of the supremum value. If the supremum is also the maximum, `bool_2` is true and false, otherwise.

`ppl_Polyhedron_maximize_with_point handle lin_expr`

Returns a record `bool_1 * coefficient_1 * coefficient_2 * bool_2 * Point` `bool_1` is true if the polyhedron P referenced by `handle` is not empty and `lin_expr` is bounded from above in P and false, otherwise. `coefficient_1` is the numerator of the supremum value and `coefficient_2` the denominator of the supremum value. If the supremum is also the maximum, `bool_2` is true and false, otherwise. `Point` is the point or closure point where `lin_expr` reaches the supremum.

`ppl_Polyhedron_minimize handle lin_expr`

Returns a record `bool_1 * coefficient_1 * coefficient_2 * bool_2` `bool_1` is true if the polyhedron P referenced by `handle` is not empty and `lin_expr` is bounded from below in P and false, otherwise. `coefficient_1` is the numerator of the infinum value and `coefficient_2` the denominator of the infinum value. If the infinum is also the minimum, `bool_2` is true and false, otherwise.

`ppl_Polyhedron_minimize_with_point handle lin_expr`

Returns a record `bool_1 * coefficient_1 * coefficient_2 * bool_2` `bool_1` is true if the polyhedron P referenced by `handle` is not empty and `lin_expr` is bounded from below in P and false, otherwise. `coefficient_1` is the numerator of the infinum value and `coefficient_2` the denominator of the infinum value. If the infinum is also the minimum, `bool_2` is true and false, otherwise. `Point` is the point or closure point where `lin_expr` reaches the infinum.

`ppl_Polyhedron_is_topologically_closed handle`

Returns true if the polyhedron referenced by `handle` is topologically closed and false, otherwise.

ppl_Polyhedron_contains_Polyhedron handle_1 handle_2

Returns true if the polyhedron referenced by handle_2 is included in or equal to the polyhedron referenced by handle_1 and false, otherwise.

ppl_Polyhedron_strictly_contains_Polyhedron handle_1 handle_2

Returns true if the polyhedron referenced by handle_2 is included in but not equal to the polyhedron referenced by handle_1 and false, otherwise.

ppl_Polyhedron_is_disjoint_from_Polyhedron handle_1 handle_2

Returns true if the polyhedron referenced by handle_1 is disjoint from the polyhedron referenced by handle_2 and false, otherwise.

ppl_Polyhedron_equals_Polyhedron handle_1 handle_2

Returns true if the polyhedron referenced by handle_1 is equal to the polyhedron referenced by handle_2 and false, otherwise.

ppl_Polyhedron_OK handle

Returns true if the polyhedron referenced by handle is well formed, i.e., if it satisfies all its implementation invariants and false, otherwise. Useful for debugging purposes.

ppl_Polyhedron_add_constraint handle constraint

Updates the polyhedron referenced by handle to one obtained by adding constraint to its constraint system.

ppl_Polyhedron_add_generator handle generator

Updates the polyhedron referenced by handle to one obtained by adding generator to its generator system.

ppl_Polyhedron_add_constraints handle constraint_system

Updates the polyhedron referenced by handle to one obtained by adding to its constraint system the constraints in constraint_system.

ppl_C_Polyhedron_add_generators handle generator_system

Updates the polyhedron referenced by handle to one obtained by adding to its generator system the generators in generator_system.

`ppl_Polyhedron_intersection_assign handle_1 handle_2`

Assigns to the polyhedron referenced by `handle_1` its intersection with the polyhedron referenced by `handle_2`.

`ppl_Polyhedron_poly_hull_assign handle_1 handle_2`

Assigns to the polyhedron referenced by `handle_1` its poly-hull with the polyhedron referenced by `handle_2`.

`ppl_Polyhedron_poly_difference_assign handle_1 handle_2`

Assigns to the polyhedron referenced by `handle_1` its poly-difference with the polyhedron referenced by `handle_2`.

`ppl_Polyhedron_affine_image handle var lin_expr coefficient`

Transforms the polyhedron referenced by `handle` assigning the affine expression `lin_expr/coefficient` to `var`.

`ppl_Polyhedron_affine_preimage handle var lin_expr coefficient`

This is the inverse transformation to that for `ppl_affine_image`.

`ppl_Polyhedron_bounded_affine_image handle var lin_expr_1 lin_expr_2 coefficient`

Transforms the polyhedron referenced by `handle` assigning the image with respect to the transfer relation `lin_expr_1/coefficient <= var <= lin_expr_2/coefficient`.

`ppl_Polyhedron_generalized_affine_image handle var Relation_Symbol lin_expr coefficient`

Transforms the polyhedron referenced by `handle` assigning the generalized affine image with respect to the transfer function `var Relation_Symbol lin_expr/coefficient`.

`ppl_Polyhedron_generalized_affine_image_lhs_rhs handle lin_expr_1 Relation_Symbol lin_expr_2`

Transforms the polyhedron referenced by `handle` assigning the generalized affine image with respect to the transfer function `lin_expr_1 Relation_Symbol lin_expr_2`.

`ppl_Polyhedron_time_elapse_assign handle_1 handle_2`

Assigns to the polyhedron $\mathcal{P}$ referenced by `handle_1` the time-elapse ($\mathcal{P} \nearrow \mathcal{Q}$) with the polyhedron $\mathcal{Q}$ referenced by `handle_2`.

`ppl_Polyhedron_BHRZ03_widening_assign handle_1 handle_2`

If the polyhedron $\mathcal{P}_1$ referenced by `handle_1` contains the polyhedron $\mathcal{P}_2$ referenced by `handle_2`, then `handle_1` will refer to the BHRZ03-widening of $\mathcal{P}_1$ with $\mathcal{P}_2$.

`ppl_Polyhedron_BHRZ03_widening_assign_with_tokens handle_1 handle_2 c_unsigned_1`

It is assumed that the polyhedron $\mathcal{P}_1$ referenced by `handle_1` contains the polyhedron $\mathcal{P}_2$ referenced by `handle_2`; let $\mathcal{P}$ denote the BHRZ03-widening of $\mathcal{P}_1$ with $\mathcal{P}_2$, Assuming that the quantity t_1 given by `c_unsigned_1` is the number of tokens available, Then this function will return the number of tokens remaining at the end of the operation.

`ppl_Polyhedron_limited_BHRZ03_extrapolation_assign handle_1 handle_2 constraint_system`

If the polyhedron $\mathcal{P}_1$ referenced by `handle_1` contains the polyhedron $\mathcal{P}_2$ referenced by `handle_-2`, then `handle_1` will refer to the BHRZ03-extrapolation of $\mathcal{P}_1$ with $\mathcal{P}_2$ improved by enforcing the constraints in `constraint_system`.

`ppl_Polyhedron_limited_BHRZ03_extrapolation_assign_with_tokens handle_1 handle_2 constraint_system c_unsigned_1`

It is assumed that the polyhedron $\mathcal{P}_1$ referenced by `handle_1` contains the polyhedron $\mathcal{P}_2$ referenced by `handle_2`; let $\mathcal{P}$ denote the BHRZ03-extrapolation of $\mathcal{P}_1$ with $\mathcal{P}_2$, improved by enforcing those constraints in `constraint_system`.

Assuming that the quantity t_1 given by `c_unsigned_1` is the number of tokens available, then this function will return the number of tokens t_2 remaining at the end of the operation.

`ppl_Polyhedron_bounded_BHRZ03_extrapolation_assign handle_1 handle_2 constraint_system`

If the polyhedron $\mathcal{P}_1$ referenced by `handle_1` contains the polyhedron $\mathcal{P}_2$ referenced by `handle_-2`, then `handle_1` will refer to the BHRZ03-extrapolation of $\mathcal{P}_1$ with $\mathcal{P}_2$ improved by enforcing the constraints in `constraint_system` together with all constraints of the form $\pm x \leq r$ and $\pm x < r$ that are satisfied by every point in $\mathcal{P}_1$.

`ppl_Polyhedron_bounded_BHRZ03_extrapolation_assign_with_tokens handle_1 handle_2 constraint_system c_unsigned_1`

It is assumed that the polyhedron $\mathcal{P}_1$ referenced by `handle_1` contains the polyhedron $\mathcal{P}_2$ referenced by `handle_2`; let $\mathcal{P}$ denote the BHRZ03-extrapolation of $\mathcal{P}_1$ with $\mathcal{P}_2$ improved by enforcing those constraints in `constraint_system` together with all constraints of the form $\pm x \leq r$ and $\pm x < r$ that are satisfied by every point in $\mathcal{P}_1$.

Assuming that the quantity t_1 given by `c_unsigned_1` is the number of tokens available, this function will return the number of tokens t_2 remaining at the end of the operation.

`ppl_Polyhedron_H79_widening_assign handle_1 handle_2`

If the polyhedron $\mathcal{P}_1$ referenced by `handle_1` contains the polyhedron $\mathcal{P}_2$ referenced by `handle_2`, then `handle_1` will refer to the H79-widening of $\mathcal{P}_1$ with $\mathcal{P}_2$.

`ppl_Polyhedron_H79_widening_assign_with_tokens handle_1 handle_2 c_-unsigned_1`

It is assumed that the polyhedron $\mathcal{P}_1$ referenced by `handle_1` contains the polyhedron $\mathcal{P}_2$ referenced by `handle_2`; let $\mathcal{P}$ denote the H79-widening of $\mathcal{P}_1$ with $\mathcal{P}_2$, Assuming that the quantity t_1 given by `c_unsigned_1` is the number of tokens available, Then this function will return the number of tokens remaining at the end of the operation.

`ppl_Polyhedron_limited_H79_extrapolation_assign handle_1 handle_2 constraint_system`

If the polyhedron $\mathcal{P}_1$ referenced by `handle_1` contains the polyhedron $\mathcal{P}_2$ referenced by `handle_2`, then `handle_1` will refer to the H79-extrapolation of $\mathcal{P}_1$ with $\mathcal{P}_2$ improved by enforcing the constraints in `constraint_system`.

`ppl_Polyhedron_limited_H79_extrapolation_assign_with_tokens handle_1 handle_2 constraint_system c_unsigned_1`

It is assumed that the polyhedron $\mathcal{P}_1$ referenced by `handle_1` contains the polyhedron $\mathcal{P}_2$ referenced by `handle_2`; let $\mathcal{P}$ denote the H79-extrapolation of $\mathcal{P}_1$ with $\mathcal{P}_2$, improved by enforcing those constraints in `constraint_system`.

Assuming that the quantity t_1 given by `c_unsigned_1` is the number of tokens available, then this function will return the number of tokens t_2 remaining at the end of the operation.

`ppl_Polyhedron_bounded_H79_extrapolation_assign handle_1 handle_2 constraint_system`

If the polyhedron $\mathcal{P}_1$ referenced by `handle_1` contains the polyhedron $\mathcal{P}_2$ referenced by `handle_2`, then `handle_1` will refer to the H79-extrapolation of $\mathcal{P}_1$ with $\mathcal{P}_2$ improved by enforcing the constraints in `constraint_system` together with all constraints of the form $\pm x \leq r$ and $\pm x < r$ that are satisfied by every point in $\mathcal{P}_1$.

`ppl_Polyhedron_bounded_H79_extrapolation_assign_with_tokens handle_1 handle_2 constraint_system c_unsigned_1`

It is assumed that the polyhedron $\mathcal{P}_1$ referenced by `handle_1` contains the polyhedron $\mathcal{P}_2$ referenced by `handle_2`; let $\mathcal{P}$ denote the H79-extrapolation of $\mathcal{P}_1$ with $\mathcal{P}_2$, improved by enforcing those constraints in `constraint_system` together with all constraints of the form $\pm x \leq r$ and $\pm x < r$ that are satisfied by every point in $\mathcal{P}_1$.

Assuming that the quantity t_1 given by `c_unsigned_1` is the number of tokens available, this function will return the number of tokens t_2 remaining at the end of the operation.

`ppl_Polyhedron_topological_closure_assign handle`

Assigns to the polyhedron referenced by `handle` its topological closure.

`ppl_Polyhedron_add_space_dimensions_and_embed handle space_dimension`

Embeds the polyhedron referenced by `handle` in a space that is enlarged by `space_dimension` dimensions,

`ppl_Polyhedron_concatenate_assign handle_1 handle_2`

Updates the polyhedron $\mathcal{P}_1$ referenced by `handle_1` by first embedding $\mathcal{P}_1$ in a new space enlarged by the space dimensions of the polyhedron $\mathcal{P}_2$ referenced by `handle_2`, and then adds to its system of constraints a renamed-apart version of the constraints of $\mathcal{P}_2$.

`ppl_Polyhedron_add_space_dimensions_and_project handle space_dimension`

Projects the polyhedron referenced by `handle` onto a space that is enlarged by `space_dimension` dimensions,

`ppl_Polyhedron_remove_space_dimensions handle Int_List`

Removes the space dimensions given by the identifiers of the PPL variables in list `Int_List` from the polyhedron referenced by `handle`. The identifiers for the remaining PPL variables are renumbered so that they are consecutive and the maximum index is less than the number of dimensions.

`ppl_Polyhedron_remove_higher_space_dimensions handle space_dimension`

Projects the polyhedron referenced to by `handle` onto the first `space_dimension` dimensions.

`ppl_Polyhedron_expand_space_dimension handle var space_dimension`

`space_dimension` copies of the space dimension referenced by variable `var` are added to the polyhedron referenced to by `handle`.

`ppl_Polyhedron_fold_space_dimensions handle list_of_vars var`

The space dimensions referenced by the PPL variables in list `list_of_vars` are folded into the dimension referenced by `var` and removed. The result is undefined if `list_of_vars` does not have the properties described in Section *Folding Multiple Dimensions of the Vector Space into One Dimension* of the main PPL user manual.

`ppl_Polyhedron_map_space_dimensions handle p_func`

Maps the space dimensions of the polyhedron referenced by `handle` using the partial function defined by a list of pairs of integers `p_func`. The result is undefined if `p_func` does not encode a partial function with the properties described in Section *Mapping the Dimensions of the Vector Space* of the main PPL user manual.

2 Module Ppl_ocaml_globals

```
type degenerate_element =
  | Universe
  | Empty
```

```
type linear_expression =
  | Variable of int
  | Coefficient of Gmp.Z.t
  | Unary_Plus of linear_expression
  | Unary_Minus of linear_expression
  | Plus of linear_expression * linear_expression
  | Minus of linear_expression * linear_expression
  | Times of Gmp.Z.t * linear_expression
```

```
type linear_constraint =
  | Less_Than of linear_expression * linear_expression
  | Less_Or_Equal of linear_expression * linear_expression
  | Equal of linear_expression * linear_expression
  | Greater_Than of linear_expression * linear_expression
  | Greater_Or_Equal of linear_expression * linear_expression
```

```
type linear_generator =
  | Line of linear_expression
  | Ray of linear_expression
  | Point of linear_expression * Gmp.Z.t
  | Closure_Point of linear_expression * Gmp.Z.t
```

```
type linear_grid_generator =
  | Grid_Line of linear_expression
  | Grid_Parameter of linear_expression * Gmp.Z.t
  | Grid_Point of linear_expression * Gmp.Z.t
```

```
type poly_gen_relation =
  | Subsumes
```

```
type poly_con_relation =
  | Is_Disjoint
  | Strictly_Intersects
  | Is_Included
  | Saturates
```

```
type relation_with_congruence =
  | Is_Disjoint
  | Strictly_Intersects
  | Is_Included
```

```
type linear_congruence = linear_expression * linear_expression *
  Gmp.Z.t
```

```
type constraint_system = linear_constraint list
```

```
type generator_system = linear_generator list
```

```
type grid_generator_system = linear_grid_generator list
```

```
type congruence_system = linear_congruence list
```

```
type relation_symbol =
  | Less_Than_RS
  | Less_Or_Equal_RS
  | Equal_RS
```

```
  | Greater_Than_RS
  | Greater_Or_Equal_RS
type complexity_class =
  | Polynomial_Complexity
  | Simplex_Complexity
  | Any_Complexity
type optimization_mode =
  | Minimization
  | Maximization
type mip_problem_status =
  | Unfeasible_Mip_Problem
  | Unbounded_Mip_Problem
  | Optimized_Mip_Problem
type control_parameter_name =
  | Pricing
type control_parameter_value =
  | Pricing_Steepest_Edge_Float
  | Pricing_Steepest_Edge_Exact
  | Pricing_Textbook
val ppl_version_major : unit -> int
val ppl_version_minor : unit -> int
val ppl_version_revision : unit -> int
val ppl_version_beta : unit -> int
val ppl_version : unit -> string
val ppl_banner : unit -> string
val ppl_set_rounding_for_PPL : unit -> unit
val ppl_restore_pre_PPL_rounding : unit -> unit
type mip_problem
val ppl_new_MIP_Problem_from_space_dimension : int -> mip_problem
val ppl_new_MIP_Problem :
  int ->
  constraint_system ->
  linear_expression ->
  optimization_mode -> mip_problem
val ppl_MIP_Problem_space_dimension : mip_problem -> int
val ppl_MIP_Problem_constraints : mip_problem -> constraint_system
val ppl_MIP_Problem_add_space_dimensions_and_embed :
  mip_problem -> int -> unit
val ppl_MIP_Problem_add_to_integer_space_dimensions :
  mip_problem -> int list -> unit
val ppl_MIP_Problem_add_constraint : mip_problem -> linear_constraint -> unit
val ppl_MIP_Problem_add_constraints :
  mip_problem -> constraint_system -> unit
val ppl_MIP_Problem_set_objective_function :
  mip_problem -> linear_expression -> unit
val ppl_MIP_Problem_is_satisfiable : mip_problem -> bool
val ppl_MIP_Problem_solve : mip_problem -> mip_problem_status
val ppl_MIP_Problem_optimization_mode : mip_problem -> optimization_mode
```

```
val ppl_MIP_Problem_feasible_point : mip_problem -> linear_generator
val ppl_MIP_Problem_optimizing_point : mip_problem -> linear_generator
val ppl_MIP_Problem_objective_function : mip_problem -> linear_expression
val ppl_MIP_Problem_optimal_value : mip_problem -> Gmp.Z.t * Gmp.Z.t
val ppl_MIP_Problem_evaluate_objective_function :
  mip_problem ->
  linear_generator -> Gmp.Z.t * Gmp.Z.t
val ppl_MIP_Problem_OK : mip_problem -> bool
val ppl_MIP_Problem_clear : mip_problem -> unit
val ppl_MIP_Problem_set_optimization_mode :
  mip_problem -> optimization_mode -> unit
val ppl_MIP_Problem_set_control_parameter :
  mip_problem ->
  control_parameter_value -> unit
val ppl_MIP_Problem_get_control_parameter :
  mip_problem ->
  control_parameter_name ->
  control_parameter_value
val ppl_MIP_Problem_swap : mip_problem -> mip_problem -> unit
```

Compilation and Installation

When the Parma Polyhedra Library is configured, it tests for the existence of the OCaml system. If OCaml is correctly installed in a standard location, things are arranged so that the OCaml interface is built and installed.

3 GNU General Public License

Version 3, 29 June 2007

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```
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```

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5 Module Index

5.1 Modules

Here is a list of all modules:

6 Module Documentation

6.1 OCaml Language Interface

The Parma Polyhedra Library comes equipped with an interface for the OCaml language.

Index